Start from London Equation

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If the electronic system interacts with radiation field. the momentum $\mathbf{p}$ is replaced by

$$\mathbf{p} \rightarrow \mathbf{p} - \frac{e}{c}\mathbf{A} \tag{1}$$

where $\mathbf{A}$ is called the vector potential and considered to be the source of the electro-magnetic field, and e is the coupling constant. This is so-called the minimal interaction hypothesis.

Then current becomes

$$\mathbf{j} = -\frac{i}{2}(\phi^*\nabla\phi - \phi\nabla\phi^*) - |\phi|^2\mathbf{A} \quad (c = 1,\ e = 1) \tag{2}$$

The first term is called the paramagnetic current, while the second term, the diamagnetic current. The diamagnetic susceptibility in ring compounds is originated from the secodly induced diamagnetic current due to the vector potential $\mathbf{A}$.

London also asserted that in the superconducting state, the wave function of the first term is rigid, hardly cause the paramagnetic current, and the diamagnetic current survives, so thar the superconducting current is diamagnetic. Fowever this assertion, at first glance, is a little suspicious. Superconducting state is purely elctronic and nothing to do with the external field.

We have to seek for internally $\mathbf{A}$ so long as what London has said is true.

Let the Nambu Spinor be

$$\phi^+(x) = \begin{pmatrix} \phi^+_\uparrow(x) & \phi_\downarrow(x) \end{pmatrix} \qquad \phi(x) = \begin{pmatrix} \phi_\uparrow(x) \\ \phi^+_\downarrow(x) \end{pmatrix}. \tag{3}$$

Then we have the density matrix,

$$\rho_{rs}(x) = \begin{pmatrix} \rho^{\uparrow\uparrow}_{rs}(x) & \rho^{\uparrow\downarrow}_{rs}(x) \\ \rho^{\downarrow\uparrow}_{rs}(x) & \rho^{\downarrow\downarrow(x)}_{rs} \end{pmatrix} = \begin{pmatrix} \rho^{\uparrow}_{rs}(x) & \rho^{+}_{rs}(x) \\ \rho^{-}_{rs}(x) & \rho^{\downarrow}_{rs}(x) \end{pmatrix}. \tag{4}$$

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Any 4×4matrix is developed in terms σ matices as

$$\rho_{rs}(x) = \rho^{\mu}_{rs}(x)\sigma^{\mu}, \qquad \rho^{\mu}_{rs}(x) = \mathrm{Tr}\ \sigma^{\mu}\rho_{rs}(x) \tag{5}$$

and tben

$$H = (\bar{\mathbf{a}}_r\mathbf{a}_s)\sigma^{\mu}h^{\mu}_{rs} + (\bar{\mathbf{a}}_r\mathbf{a}_s)\sigma^{\mu}(\rho^{\mu}_{rs};\rho^{\nu}_{tu})\sigma^{\nu}\bar{\mathbf{a}}_t\mathbf{a}_u \tag{6}$$

We seek for the vector potential hidden in the electron- electron interaction. To this end we observe the electronic energy of the system. We now find the vector potential is hidden in the electron-electron interaction.

$$\begin{aligned} A^3(x) &= \int dx_2\rho^3_{rs}(x_)v(x_-x_2)\rho^3_{tu}(x_2)\sigma^3(\bar{\mathbf{a}}_t\mathbf{a}_u) \\ A^+(x) &= \int dx_2\rho^+_{rs}(x_)v(x_-x_2)\rho^-_{tu}(x_2)\sigma^-(\bar{\mathbf{a}}_t\mathbf{a}_u) \\ A^-(x) &= \int dx_2\rho^-_{rs}(x_)v(x_-x_2)\rho^+_{tu}(x_2)\sigma^-(\bar{\mathbf{a}}_t\mathbf{a}_u). \end{aligned} \tag{7}$$

We now write the electron-electron interaction as the gauge factor over the wave function,

$$\phi(x) \to \phi(x)e^{-(e/c)dx^i A^i(x)} \quad i = 3, +, - \tag{8}$$

However a queation remains. In getting the expectation value we take Tr operation which requires for the indices matrixelemrnts $\sigma^{\mu}\sigma^{\nu} \to \sigma^0$. In this case, the non-relativistic treatment for the off-diagonal elements reduce to vanishing upon integration over the spin space. Precisely spraking. the elcctron-electron interaction $v(x-x')$ is spacialy symmetric, while the densities $\rho^+(x)$ and $\rho^-(x')$ are mutually orthogonal in the spin space, so that the integral in qestion vanishes upon the spin space integration. This difficulty is avoided by BCS wirh the electron-phonon interaction and by Bogoliubov with the coherent interation. Our treatment prefers to the relativitic consideration, say the spin-orbit coupling in which the spin operator rotates the spin space.